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The Role of Observational Errors in Optimum Interpolation Analysis

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1. Introduction

With the advent of new observing systems and the approach of the First GARP Global Experiment (FGGE), it appears desirable for data providers to have some knowledge of how their product is to be used in the statistical "optimum interpolation" analysis schemes which will operate at most of the Level III centers during FGGE. It is the hope of the author that this paper will serve as a source of information on the way observational data is handled by optimum interpolation analysis schemes.

The theory of optimum interpolation analysis, especially with regard to the role of observational errors, is reviewed in Section 2. In Section 3, some comments about determining observational error characteristics are given along with examples of same. Section 4 discusses error-checking procedures which are used to identify suspect observations. These latter procedures are an integral part of any analysis scheme.

2. Optimum Interpolation Analysis

The job of the analyst is to take an irregular distribution of observations of variable quality and obtain the best possible estimate of a meteorological field at a regular network of grid points. The optimum interpolation analysis scheme attempts to accomplish this goal by minimizing the mean square interpolation error for a large ensemble of analysis situations. The method is best described in the literature by Gandin (1963) and Gandin and Kagan (1974). Applications of optimum interpolation to multivariate analysis problems are given by Schlatter (1975), Bergman (1976), and Rutherford (1976). The univariate theory is presented below, but the extension to the multivariate case is straightforward insofar as observational errors are concerned.

Consider a model grid point and level (which may be an isobaric, isentropic, or isohypsic level, or the midpoint of a model "sigma-layer") and the observations contained in a specified neighborhood volume about the grid point. This volume is usually no greater than 15° latitude horizontal radius and half the total atmosphere in pressure-thickness. Let F be a meteorological field which is being analyzed. We wish to estimate the true value F_a which applies at grid point/level "a" from the n observed values $\hat{F}_i$ in its vicinity and a reasonably good "guess" value $\tilde{F}_a$. The "guess" is usually a short-period forecast, persistence, climatology, or some blend of these.

The procedure is to interpolate the gridded field $\tilde{F}$ to the locations of the observations by a simple method. The difference

$$\hat{f}_i \equiv \hat{F}_i - \tilde{F}_i; \; i = 1,n \tag{1}$$

between the observed and guessed values at the location of each observation is computed. These $\hat{f}_i$ are commonly referred to as "residuals." We recognize that the residual of the i^{th} observation consists of two components,

$$\hat{f}_i = f_i + \varepsilon_i, \tag{2}$$

where f_i is the difference between the true value F_i and the guess value $\tilde{F}_i$ (the "true residual," in contrast to the "observed residual" $\hat{f}_i$), and ε_i is the observational error.

The analyzed value $\hat{F}_a$ at the particular grid point and level considered is then obtained by adding a weighted linear sum of the observed residuals to the guess value $\tilde{F}_a$,

$$\hat{F}_a = \tilde{F}_a + \sum_{i=1}^{n} c_i \hat{f}_i$$

$$= \tilde{F}_a + \sum_{i=1}^{n} c\ (f_i + \varepsilon_i), \tag{3}$$

where c_i is the weight to be assigned to the residual of the i^{th} observation.

Depending on how these weights are determined, the mean square error of the analyzed value $\hat{F}_a$ for a large ensemble of analyses is given by

$$\overline{E^2} \equiv \overline{(F_a - \hat{F}_a)^2} = \overline{[F_a - \tilde{F}_a - \sum_{i=1}^{n} c_i(f_i + \varepsilon_i)]^2} \tag{4}$$

The statistical optimum interpolation scheme requires that the weights be chosen so that $\overline{E^2}$ is a minimum. Hence, differentiating $\overline{E^2}$ partially with respect to each of the c_i and equating to zero leads to the following set of equations:

$$\sum_{j=1}^{n} \overline{(f_i + \varepsilon_i)(f_j + \varepsilon_j)}c_j = \overline{f_a(f_i + \varepsilon_i)}; \quad i = 1,2,\ldots,n$$

or, expanding the product terms,

$$\sum_{j=1}^{n} \left(\overline{f_i f_j} + \overline{f_i \varepsilon_j} + \overline{f_j \varepsilon_i} + \overline{\varepsilon_i \varepsilon_j}\right)c_j = \overline{f_a f_i} + \overline{f_a \varepsilon_i}\ ; \quad i = 1,2,\ldots,n \tag{5}$$

These linear equations may be solved for the c_i provided the quantities $\overline{f_i f_j}$, $\overline{f_i \varepsilon_j}$, etc., can be specified. (These quantities are covariances only if the $\overline{f_i}$ and $\overline{\varepsilon_i}$ are zero, strictly speaking.)

In actually carrying out the computations, it is convenient to express this set of equations in the following normalized form:

$$\sum_{j=1}^{n}\left(\mu_{ij} + \tau_{ij}\sigma_j + \tau_{ji}\sigma_i + \rho_i\ \sigma_i\sigma_j\right)c_j' = \mu_{ai} + \tau_{ai}\sigma_i \ ; \quad i = 1,2,\ldots,n \qquad (6)$$

where
$$\mu_{ij} \equiv \overline{f_i f_j}/\left(\overline{f_i^2}\,\overline{f_j^2}\right)^{\frac{1}{2}}$$
$$\tau_{ij} \equiv \overline{f_i \varepsilon_j}/\left(\overline{f_i^2}\,\overline{\varepsilon_j^2}\right)^{\frac{1}{2}}$$
$$\rho_{ij} \equiv \overline{\varepsilon_i \varepsilon_j}/\left(\overline{\varepsilon_i^2}\,\overline{\varepsilon_j^2}\right)^{\frac{1}{2}}$$
$$\sigma_i \equiv \left(\overline{\varepsilon_i^2}/\overline{f_i^2}\right)^{\frac{1}{2}}$$
and
$$c_j' \equiv \left(\overline{f_i^2}/\overline{f_a^2}\right)^{\frac{1}{2}} c_j \ .$$

The first of these terms, μ_{ij}, is the correlation of the true residual at the i^{th} observational location with that at the j^{th} location. It also appears on the right side of (6) as μ_{ai}, the correlation between the true residual at the grid point/level (which is what we are trying to estimate as closely as possible) and that at the i^{th} observational location. This correlation is a function of location only and it is obviously dependent on the characteristics of the guess field $\tilde{F}$ as well as the true field F. There is a considerable literature on the determination of this correlation, for example Thiebaux (1975, 1976), Julian and Thiebaux (1975), Hollett (1975), and Bergman (1977). In currently operational optimum interpolation analysis schemes, various simplifying assumptions are made about the nature of the μ correlation, and it is represented by an analytical function of the distance separating the two locations involved. In those schemes which use data at pressures other than the pressure at which the analyzed value is required, the three-dimensional correlation is approximated by the product of a two-dimensional isobaric correlation function and a one-dimensional correlation function which depends on the pressure difference between the two locations.

There is some controversy over whether these approximate representations of the μ correlation seriously compromise the results of the objective analysis schemes using them, see Thiebaux (1976, 1977), but the weight of current opinion is that the analysis results are relatively insensitive to the finer details of how the μ-correlation is specified (Schlatter, et al., 1977; P. Julian, personal communication). In any event, this correlation is in general dependent on the specific characteristics of the analysis/guess system being used and must be specified by the analyst. Hence it need not concern the data provider.

The second correlation, τ_{ij}, is that between the true field residual at one location with the observational error at another location. It also appears on the right side of (6) as τ_{ai}. This correlation will be non-zero for an observing system which uses the same guess field, say a 12-hour forecast, as the analysis scheme uses. An example is the minimum-retrieval method formerly used operationally to construct temperature profiles from VTPR radiance data (McMillin, et al., 1973). However, the τ-correlation may be nonzero even for other observing systems if both the observations and the guess field describe a smoothed version of reality. For example, satellite derived temperature fields typically have weaker gradients than those indicated by conventional data (Desmarais, et al., 1978), and these errors may correlate with similar errors in a forecast or climatological guess field.

The τ-correlation is a difficult term to evaluate, and to date no one has attempted to do so. All the currently operational analysis schemes assume that this correlation is uniformly zero, with the hope that observing systems will not rely strongly on a forecast or other guess field in determining their values, and that the unavoidable correlation due to data smoothing is relatively weak and unimportant compared to the other correlations.

The third correlation, ρ_{ij}, is that which exists between two observations at different locations. For many pairings of observations this correlation is zero. It has been demonstrated by Bergman and Bonner (1976), Schlatter and Branstator (1978), and others that errors in satellite temperature measurements from the same orbital pass are horizontally correlated, and by C. Hayden of NESS (1977, unpublished) and by Schlatter and Branstator (1978) that the same errors are correlated in the vertical for a sounding. However, the latter authors find that the vertical correlation is a weak one. Hollett (1975) has demonstrated that rawinsonde errors in height, temperature, and wind measurements are all correlated vertically to some extent. The situation for other observing systems, such as satellite winds, is unknown. As shown by Bergman and Bonner (1976), the effect of a spatial correlation of observational errors is to reduce the amount of independent information that the observations provide in a univariate analysis. On the other hand, Seaman (1977) has shown that gradient information in multivariate analysis is preserved by the presence of spatially correlated errors. An example of the latter case is the use of geopotential height observations in the analysis of the wind field with an assumed geostrophic wind relationship.

Finally, the normalized root-mean-square observational error, σ_i, for each of the observations appears in the set of equations (6) for determining the weights. This is simply the ratio of the RMS observational error to the RMS "true residual" (essentially the RMS error of the guess field). The former is a property of the observing system and, if the individual errors are known, may vary from one observation to another within a set of observations all from the same observing system.

Substitution of (5) in (4) and normalizing the resulting equation with respect to $\overline{f_a^2}$ yields the following expression for the normalized estimated mean square analysis error:

$$\overline{\sigma_a^2} = 1 - \sum_{i=1}^{n} c_i' \left(\mu_{ai} + \tau_{ai}\sigma_i\right), \tag{7}$$

where

$$\sigma_a^2 \equiv \overline{E^2}/\overline{f_a^2}.$$

Although the error correlations ρ_{ij} do not appear explicitly in (7), nevertheless the weights c_i' as determined by (6) are a function of the ρ_{ij}. Therefore, the analysis error σ_a^2 is an implicit function of the ρ_{ij}. Similarly, the weights c_i' are a function of the observational errors σ_i even when the τ_{ai} are zero, consequently σ_a^2 is implicitly dependent on the observational errors σ_i.

The following simple example illustrates the dependence of analysis error on the observational error characteristics. Following Gandin, et al. (1972), consider three observations which are equidistant from a grid point and also from each other. Hence the three observations from an equilateral triangle of length h on a side, where $h/\sqrt{3}$ is the distance between the centrally located grid point and each observation. Assume that the observations all have the same error σ, that the correlations $\mu(s)$ and $\rho(s)$ are functions of separation distance only, and that the τ correlation can be neglected. Then the weights of all three observations are the same and are given by

$$c_i' = \frac{\mu(h/\sqrt{3})}{1 + 2\mu(h) + \sigma^2[1 + 2\rho(h)]} \tag{8}$$

and the normalized mean-square-analysis error by

$$\sigma_a^2 = 1 - \frac{3\mu^2(h/\sqrt{3})}{1 + 2\mu(h) + \sigma^2[1 + 2\rho(h)]} \tag{9}$$

Thus, for this case, the analysis error increases as the observational error σ increases, as would be expected. This result is generally true for any number and distribution of observations. Additionally, the magnitude of σ_a increases as the error correlation ρ increases, especially if the error σ is relatively large. This result is also generally true for univariate analyses and is a reflection of the loss of independent primary information provided by observations whose errors are correlated. On the other hand, information derived from the difference between a pair of observations, such as the gradient of the mass field, will have a smaller effective error when the individual errors of the observations are correlated. Thus, for example, geopotential heights with spatially correlated errors will result in larger errors in the analyzed heights, but smaller errors in analyzed height gradients, than will the same data with uncorrelated errors.

From the foregoing, it is obviously important that both the observational errors and the error correlations should be specified with reasonable accuracy if the observations are to receive their correct weights in the analysis. Only in this way will the analysis error actually be close to the theoretical minimum that is implied by the particular types and distributions of the observations used for the analysis. This is so because the minimization of (4) implies that any assignment of observational weights other than those implied by (6) will always result in a larger value of analysis error as given by (7).

3. Determination of Observational Error Characteristics

First, the observational error ε will be discussed, and then the correlation of observational errors ρ will be taken up. In both cases, the determination of these quantities is beset with difficulties.

a. Observational Errors

At least four methods have been used to determine the statistical observational error levels of various observing systems. These are:

(1) Theoretical and laboratory estimates of instrumental, transmission, recording, and interpretative errors involved in obtaining the observation.

(2) Extrapolation of the variance of the difference between pairs of separated observations to zero separation.

(3) Comparisons with other observations whose error level is either known or assumed to be small.

(4) Comparison with an analysis, preferably constructed from other types of observations than those whose error is being determined.

There may be other methods which have been tried or which have potential.

The first of these is frequently considered to be an underestimate of the observational error appropriate for use in objective analysis. Primarily, this is because the sub-grid-scale features of atmospheric flow which may be measured by the observing system must be considered as "noise" superimposed on a gridded objective analysis. This noise becomes part of the random error of the observation. In this respect, an observing system which has the same degree of smoothness and resolution as that of the analysis grid would be preferred. Note that this comment refers not to the spacing of observations, but rather to the volume sampled by a single observation. The difference between the observational error at a point and the observational error appropriate to a given resolution is illustrated by a study of Bruce, et al. (1977). In this study, the variances of the observed temperature difference between pairs of closely spaced radiosonde soundings at White Sands Missile Range, New Mexico, are computed. Extrapolation of these variances, plotted as a function of separation distance, to zero separation gives an estimate of the observational error at a point. The resulting estimates of error standard deviation range from 0.5° to 1.0°C, depending on altitude. Using linear interpolation and areal averaging, Bruce, et al., then proceed to demonstrate that the expected difference between a point radiosonde temperature measurement and the corresponding mean temperature for a 100 km square area is between 1.0 and 1.4°C. The reader is referred to the paper cited above for details of the method.

The most prevalent way of assessing the error magnitudes of new observing systems is comparison with rawinsonde observations, or with an analysis constructed from them. An example for rawinsonde vs. satellite VTPR temperatures at mandatory levels is shown as Fig. 1 (Courtesy of Product Systems Branch, Office of Operations, National Environmental Satellite Service). Direct comparisons of new observing systems with rawinsonde observations are made provided the pair of observations being compared are located within a space/time "window" of each other. The window used for the comparisons of Fig. 1 is 1 degree latitude and 6 hours.

Using the notation of the preceding section, the difference between a rawinsonde and a satellite temperature observation as close together in space/time as possible is

$$\hat{T}_r - \hat{T}_s = (T_r + \varepsilon_r) - (T_s + \varepsilon_s) \quad , \tag{10}$$

and the mean square difference for a statistical sample of such comparisons is

$$\begin{aligned} \overline{(\hat{T}_r - \hat{T}_s)^2} &= \overline{(T_r - T_s + \varepsilon_r - \varepsilon_s)^2} \\ &= \overline{(T_r - T_s)^2} + 2\overline{(T_r - T_s)\varepsilon_r} - \overline{2(T_r - T_s)\varepsilon_s} \\ &\quad + \overline{\varepsilon_r^2} + \overline{\varepsilon_s^2} - \overline{2\varepsilon_r\varepsilon_s} \ . \end{aligned} \tag{11}$$

If the observational errors are assumed to be uncorrelated with each other or with the difference $(T_r - T_s)$, and if additionally the space/time window is chosen small enough so that

$$\overline{(T_r - T_s)^2} << \overline{\varepsilon_r^2} \text{ and } \overline{\varepsilon_s^2} ,$$

then

$$\overline{\varepsilon_s^2} \simeq \overline{\left(\hat{T}_r - \hat{T}_s\right)^2} - \overline{\varepsilon_r^2} . \qquad (12)$$

Thus, the mean square satellite error can be determined if an independent estimate of the rawinsonde error is available.

A critical point here is to choose the window small enough to justify neglect of the difference $(T_r - T_s)$. In the example of Fig. 1, the window allows a separation of up to 1^o latitude and 6 hours between the compared observations. Studies of the sensitivity of results to the size of the window appear to indicate that in this case the window is sufficiently small for the comparisons being made (C. Hayden, personal communication).

The same method may be used to compare observations with an analysis based on other observational data. In this case, the space part of the window problem is bypassed if the analysis is interpolated to the observational location. An independent estimate of the <u>analysis</u> error is required. If the observational and analysis errors are correlated, the result is contaminated by the presence of a non-zero term $\overline{2\varepsilon_a\varepsilon_s}$, where ε_a is the analysis error, corresponding to the term $\overline{2\varepsilon_r\varepsilon_s}$ in eqn. (11).

When comparison with an analysis is done, it is preferable to exclude the observing system whose error characteristics are being evaluated from the data base for the analysis. The magnitude of the analysis error depends, in part, on the magnitudes of the observational errors used in the analysis and on any correlation that may exist between these errors. Hence these observational errors should be independently known in order to obtain an accurate estimate of the analysis error.

Satellite (NIMBUS) thickness temperature errors have been evaluated by C. Hayden (unpublished) and by Schlatter and Branstator (1978) by means of comparison with analyses based on radiosonde data. These results agree closely with those obtained by the space/time window comparison method.

b. <u>Observational Error Correlations</u>

The author is aware of three methods that have been employed to determine the error correlation of spatially distributed observations:

(1) Space-time window comparisons with other observations which do not have correlated errors, or whose error correlation is known from other sources.

(2) Comparison with an analysis whose spatial correlation of error is known.

(3) Partitioning of observed-minus-forecast difference covariance into forecast error covariance and observational error covariance through use of empirical orthogonal functions.

The first two methods parallel methods already discussed which are used to estimate the RMS observational error. Here, we may combine the methods in one generalized treatment. Let subscript "a" refer to an observing system (or analysis) whose error characteristics are already known, and let subscript "b" refer to an observing system whose error correlation is to be evaluated. Also, let subscript 1 refer to the location of one of the b-observations and subscript 2 refer to the location of another b-observation. In the following, either the space/time windows for comparing observations a and b are assumed negligibly small, or an analyzed field value is assumed to be known at the b-observation locations. Then the covariance of the difference between observation (or analysis) a and observation b for the two locations is given by

$$\overline{\left(\hat{F}_a - \hat{F}_b\right)_1 \left(\hat{F}_a - \hat{F}_b\right)_2} = \overline{\left[\left(F_a + \varepsilon_a\right)_1 - \left(F_b + \varepsilon_b\right)_1\right]\left[\left(F_a + \varepsilon_a\right)_2 - \left(F_b + \varepsilon_b\right)_2\right]}$$

$$\begin{aligned} &= \overline{F_{a1}F_{a2}} + \overline{F_{b1}F_{b2}} - \overline{F_{a1}F_{b2}} - \overline{F_{b1}F_{a2}} \\ &\quad + \overline{F_{a1}\varepsilon_{a2}} + \overline{F_{b1}\varepsilon_{b2}} + \overline{\varepsilon_{a1}F_{a2}} + \overline{\varepsilon_{b1}F_{b2}} \\ &\quad - \overline{F_{a1}\varepsilon_{b2}} - \overline{F_{b1}\varepsilon_{a2}} - \overline{\varepsilon_{a1}F_{b2}} - \overline{\varepsilon_{b1}F_{a2}} \\ &\quad + \overline{\varepsilon_{a1}\varepsilon_{a2}} + \overline{\varepsilon_{b1}\varepsilon_{b2}} - \overline{\varepsilon_{a1}\varepsilon_{b2}} - \overline{\varepsilon_{b1}\varepsilon_{a2}} . \end{aligned} \tag{13}$$

This formal expression may be simplified by noting that:

(1) The covariance of the true field F is independent of the observing system, i.e.,

$$\overline{F_{a1}F_{a2}} = \overline{F_{b1}F_{b2}} = \overline{F_{a1}F_{b2}} = \overline{F_{b1}F_{a2}}$$

(2) The errors of different observing systems are unlikely to be correlated, i.e.,

$$\overline{\varepsilon_{a1}\varepsilon_{b2}} = \overline{\varepsilon_{b1}\varepsilon_{a2}} = 0$$

If, additionally, it is assumed that observational errors are uncorrelated with the true field values (an assumption that may not always be justified), the above expression reduces to

$$\overline{(\hat{F}_a - \hat{F}_b)_1 (\hat{F}_a - \hat{F}_b)_2} = \overline{\varepsilon_{a1}\ \varepsilon_{a2}} + \overline{\varepsilon_{b1}\ \varepsilon_{b2}}$$

or

$$\overline{\varepsilon_{b1}\varepsilon_{b2}} = \overline{\Delta\hat{F}_1\ \Delta\hat{F}_2} - \overline{\varepsilon_{a1}\varepsilon_{a2}}\ , \tag{13}$$

where

$$\Delta\hat{F}_i \equiv \left(\hat{F}_a - \hat{F}_b\right)_i\ .$$

This expression may be restated in normalized form as

$$\rho_{b1,b2} = \frac{\left(\overline{\Delta\hat{F}_1{}^2}\ \ \overline{\Delta\hat{F}_2{}^2}\right)^{1/2}}{\left(\overline{\varepsilon_{b1}^2}\ \ \overline{\varepsilon_{b2}^2}\right)^{1/2}}\ \mu_{\Delta\hat{F}_1,\Delta\hat{F}_2} - \frac{\left(\overline{\varepsilon_{a1}^2}\ \ \overline{\varepsilon_{a2}^2}\right)^{1/2}}{\left(\overline{\varepsilon_{b1}^2}\ \ \overline{\varepsilon_{b2}^2}\right)^{1/2}}\ \rho_{a1,a2}\ , \tag{14}$$

where

$$\mu_{\Delta\hat{F}_1,\Delta\hat{F}_2} \equiv \frac{\overline{\Delta\hat{F}_1\ \Delta\hat{F}_2}}{\left(\overline{\Delta\hat{F}_1{}^2}\ \ \overline{\Delta\hat{F}_2{}^2}\right)^{1/2}}$$

and the definitions of the ρ-correlations are the same as before. Thus, in addition to the observed difference correlation $\mu_{\Delta\hat{F}_1,\Delta\hat{F}_2}$, information on the correlation of the a-observation errors plus the mean-square errors of both the a- and b-observations is required in order to compute $\rho_{b1,b2}$.

The situation is simpler when $\overline{\varepsilon_{a1}\varepsilon_{a2}} = 0$. Then

$$\left(\overline{\Delta\hat{F}_i{}^2}\right) = \overline{\varepsilon_{bi}^2}$$

and

$$\rho_{b1,b2} = \mu_{\Delta\hat{F}_1,\Delta\hat{F}_2}\ . \tag{15}$$

Thus, there is a distinct advantage in using observations with uncorrelated errors as the a-observations.

Fig. 2 shows an example of isobaric correlation of satellite observed temperature errors for ten mandatory levels combined. The satellite observations were compared with rawinsonde temperatures within a space/time window of 2^o latitude and 3 hours. The rawinsonde errors were assumed to be uncorrelated isobarically. Individual correlation profiles for each of the ten isobaric levels were also computed, but they showed little variation with level.

If the b-observations are compared with an a-analysis rather than with a-observations, then the correlation $\rho_{a1,a2}$ of (14) should not automatically be assumed to be zero, even when the observations on which the analysis is based have uncorrelated errors. The errors of the analysis at two points 1 and 2 may be highly correlated for one or more of the following reasons:

(1) The analysis may start from a guess field of values which, be it persistence, a forecast, or climatology, is likely to have spatially correlated errors.

(2) In grid point objective analysis, an observation is frequently used in the analysis of more than one grid point. This may lead to correlated errors between analysis grid points even though the observational errors themselves are uncorrelated with one another.

(3) In spectral analysis schemes, the truncation of the spectral representation at some wave number results in spatially correlated analysis errors which are a function of the neglected higher frequency modes.

It may turn out that, for a particular analysis scheme and data set, the correlation $\rho_{a1,a2}$ is negligibly small compared to $\rho_{b1,b2}$; this should not be an _a priori_ assumption when doing the computations, but see below.

Computations of the isobaric correlation of temperature errors for satellite NIMBUS data have been performed by C. Hayden (unpublished) and by Schlatter and Branstator (1978) by means of comparisons with analyses based on rawinsonde data. Both of these studies assume that $\rho_{a1,b2}$ can be neglected; nevertheless, their results show reasonable agreement with the space/time window comparisons of Fig. 2.

The above-mentioned investigators have also computed the vertical correlation of NIMBUS temperature errors by comparison with analyses. The vertical correlation of analysis errors was not taken into account. The results of the two groups differ markedly, with Hayden's showing a much stronger vertical correlation of satellite errors than Schlatter and Branstator's. Presumably, this difference is related to the analyses used for the comparisons since the same satellite data (NIMBUS/DST-5) was used by both. It should be noted that space/time window comparisons with rawinsonde data do not give uncontaminated estimates of the vertical correlation of satellite errors because of the fact that rawinsonde temperatures themselves have vertically correlated errors (see below). As a result, the neglect of vertical error correlations of an analysis based on rawinsonde data is more likely to yield erroneous values for the satellite vertical error correlations than in the horizontal case.

The above methods and limitations apply as well to other observing systems whose errors are spatially correlated. Observing systems believed to have this characteristic include satellite cloud-track winds and sequential aircraft observations, but to date error correlation statistics have not been computed for these systems.

The third method, partitioning observed-minus-forecast difference covariances into observed and forecast error covariance components by means of empirical orthogonal functions, is due to Hollett (1975), and the reader is referred to this source for details of the method. Hollett has developed the method specifically for determining the vertical correlation of serial ascent rawinsonde errors.

Briefly, the method consists of expressing the observed-minus-forecast difference covariance matrix for nine mandatory levels in terms of a set of vertical eigen-functions. The covariance data may be expressed in terms of these functions. Now, the forecast errors are correlated both isobarically and vertically, but the rawinsonde errors are correlated only vertically. Thus, the degree to which each of the functions correlates isobarically can be used to partition the function into a forecast error component and an observational error component. From these partitioned functions, separate forecast error and observational error covariances can be constructed, and the observational error covariances normalized to give equivalent correlations. The results for rawinsonde wind components, heights, and temperatures are shown in Table 1.

It should be noted that the number of orthogonal functions used was limited to 9, hence separation of covariances into forecast error and observational error components was necessarily incomplete. As a result, the correlations of Table 1 are contaminated to some extent by the presence of some residual forecast error correlation. However, Hollett demonstrates that most of the separation is accounted for by the first five functions, thus the remaining contamination when using nine modes should be slight.

4. Error Checking Procedures and Quality Control

Although specification of the root-mean-square observational errors and error correlations is sufficient for the operation of an optimum interpolation analysis scheme, any operational analysis program must also include error-checking routines to guard against the inclusion of data whose individual error is markedly greater than the assumed RMS error level for the observational type.

Error checking in objective analyses is usually of two kinds:

(1) Gross error check--to eliminate obviously erroneous observations.

(2) Comparative error check--to compare an observation with its neighbors to see if it is consistent with them.

The gross error check eliminates an observation which differs by an unreasonable amount from climatology, a forecast, or other guess value. It is an easy check to make, but is one which should allow for the occasional possibility of rather large departures from climatology or from the guess value. In other words, this check should use a rather loose criterion and err in the direction of retaining bad data rather than throwing out good data. For it is precisely those observations which differ the most from a guess field or from climatological expectation that, if correct, are of most value in doing the analysis.

A comparative check ("buddy check") rejects an observation if it is in marked disagreement with neighboring observations. By the same token, it retains an observation which shows good agreement with neighbors even though the observation differs markedly from the guess field or from climatology. Various types of comparative checks are used in analysis schemes. In the Canadian operational analysis, comparative checking is done by using neighboring observations to do an optimum interpolation to the location of an observation and then comparing this value with the observed value (Rutherford, 1976). Other analysis schemes (e.g., Schlatter et al., 1976) compute the average field value from neighboring stations and compare this with the observed value.

The NMC optimum interpolation scheme compares the differences between pairs of observations with the correlation of the true field, μ_{ij}, between them. The larger the value of μ_{ij}, the smaller the difference allowed between a pair of observations. Suspect observations are flagged and are rejected if they receive two or more flags from all possible pairings. This procedure is designed to prevent bad observations from rejecting good observations in the comparative error check, a possibility that all such error checking routines must guard against.

At NMC, rawinsonde soundings are subjected to a vertical consistency check prior to their use in the analysis. Depending on the outcome of the vertical check, a rawinsonde datum is either rejected outright or assigned a relative quality indicator. This indicator is used in the error checking routines of the optimum interpolation analysis code in the following ways:

(1) In the gross error check, the higher quality data are permitted wider deviations from the guess field or climatology than are the lower quality data.

(2) In the comparative check, a higher quality observation is not permitted to be flagged as suspect by a lower quality observation. However, if both of a pair of observations have the same quality indicator, both are flagged.

For example, suppose there are six observations, three of quality "A" and three of quality "B." The observations are flagged as indicated in Table 2a, with the total number of flags for each observation on the right. Note that the flagging is mutual between A observations and between B observations, but that although A observations flag B observations, the reverse is not permitted. Observation 5 has the largest number of flags, so it is rejected first. Removing all the flags generated by observation 5 reduces the array to that shown in Table 2b. Now observation 1 has the largest number of flags, so it is rejected. The result is Table 2c. All remaining observations have less than two flags, so they are accepted for the final analysis.

The above demonstrates a way in which indicators that are only qualitative can be used in an analysis scheme. Quantitative estimates of observational error are preferable, however, since they directly affect the weights assigned and the analysis results in an optimum interpolation analysis scheme.

Finally, some remarks about error checking are offered. For some observing systems, a certain amount of checking is done by the data provider before the data arrive in the hands of the data user. It is understandable that the data provider wishes to transmit as "clean" a data set as possible, as a high percentage of bad or questionable observations may reflect unfavorably on the provider's observing system. Also, the analyst would like to be spared to some extent the task of checking large quantities of data with a high percentage of unacceptable errors. However, it should be clear from the preceding discussion that it is not desirable for the data provider to exercise too tight a control on the quality of his data. Gross error checks should have relatively wide limits to allow for the possibility of unusual (or poorly forecast) atmospheric conditions, with the proviso that analysis schemes should of necessity have gross error checks of their own. From the analyst's point of view, it is especially undesirable to have observations accepted or rejected on the basis of agreement with forecast values, since this procedure may result in a correlation of observational errors with those of the forecast.

Comparative error checking is really the province of the analyst, who is the only one in the position of having <u>all</u> the observed data available for use in the checking. When the data provider does comparative checking, it is likely to be done with an incomplete data set, which can well result in good observations being rejected. Further, there is a tendency for the data provider to make criteria for rejection too tight in order to produce what he views as a smooth, consistent set of data. This author feels that data providers should be discouraged from doing comparative error checking.

5. Summary

The basic theory of optimum interpolation analysis is presented, with emphasis on the role that observational errors have in the analysis scheme. It is shown that an estimate of the root-mean-square observational error is required, and also that the correlation between errors of observations, if nonzero, must be specified. Methods of determining these quantities from observational data statistics are discussed and examples shown. It is desirable that these statistical determinations be carried out by both the data provider and the data user. Finally, the need for observational error-checking routines which either accept or reject data is indicated. It is pointed out that the data provider should be careful not to delete good and useful data from his product when checking for errors.

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TABLE 1

Rawinsonde error correlations as a function of pressure for U,V, Z, and T from Hollett (1975).

U wind component

	850	700	500	400	300	250	200	150	100
850	1.00								
700	.25	1.00							
500	.12	.29	1.00						
400	.10	.12	.44	1.00					
300	-.00	.05	.19	.44	1.00				
250	-.02	.02	.17	.30	.56	1.00			
200	.02	.06	.15	.24	.33	.46	1.00		
150	.05	.09	.16	.16	.16	.23	.41	1.00	
100	.04	.06	.09	.13	.09	.10	.15	.21	1.00

V wind component

	850	700	500	400	300	250	200	150	100
850	1.00								
700	.23	1.00							
500	.12	.28	1.00						
400	.10	.12	.43	1.00					
300	.00	.05	.18	.43	1.00				
250	-.00	.03	.15	.29	.55	1.00			
200	.04	.07	.14	.22	.31	.43	1.00		
150	.06	.11	.14	.14	.13	.19	.36	1.00	
100	.06	.08	.11	.10	.05	.05	.12	.22	1.00

Z (geopotential height)

	850	700	500	400	300	250	200	150	100
850	1.00								
700	.72	1.00							
500	.56	.74	1.00						
400	.48	.69	.91	1.00					
300	.45	.63	.84	.92	1.00				
250	.42	.60	.80	.85	.94	1.00			
200	.38	.56	.76	.79	.88	.96	1.00		
150	.38	.56	.76	.80	.87	.92	.96	1.00	
100	.41	.56	.71	.76	.79	.75	.74	.83	1.00

T (temperature)

	850	700	500	400	300	250	200	150	100
850	1.00								
700	.91	1.00							
500	-.21	1.26	1.00						
400	.05	-.30	.76	1.00					
300	.08	-.47	.27	.20	1.00				
250	-.00	.25	-.24	-.12	.22	1.00			
200	.04	-.10	.15	.17	-.15	-.10	1.00		
150	-.01	.01	.08	.18	.27	.29	.38	1.00	
100	-.02	-.16	-.12	-.11	.50	.77	.21	.22	1.00

Table 2

Example of Use of Flags in Comparative Data Check.

a. Flags generated by data comparisons:

	1	2	3	4	5	6	Total
1 (A)		F	F				2
2 (A)	F						1
3 (A)	F						1
4 (B)		F					1
5 (B)	F	F				F	3
6 (B)	F				F		2

b. Flags after deletion of observation 5:

	1	2	3	4	6	Total
1 (A)		F	F			2
2 (A)	F					1
3 (A)	F					1
4 (B)		F				1
6 (B)	F					1

c. Flags after deletion of observation 1:

	2	3	4	6	Total
2 (A)					0
3 (A)					0
4 (B)	F				1
6 (B)					0

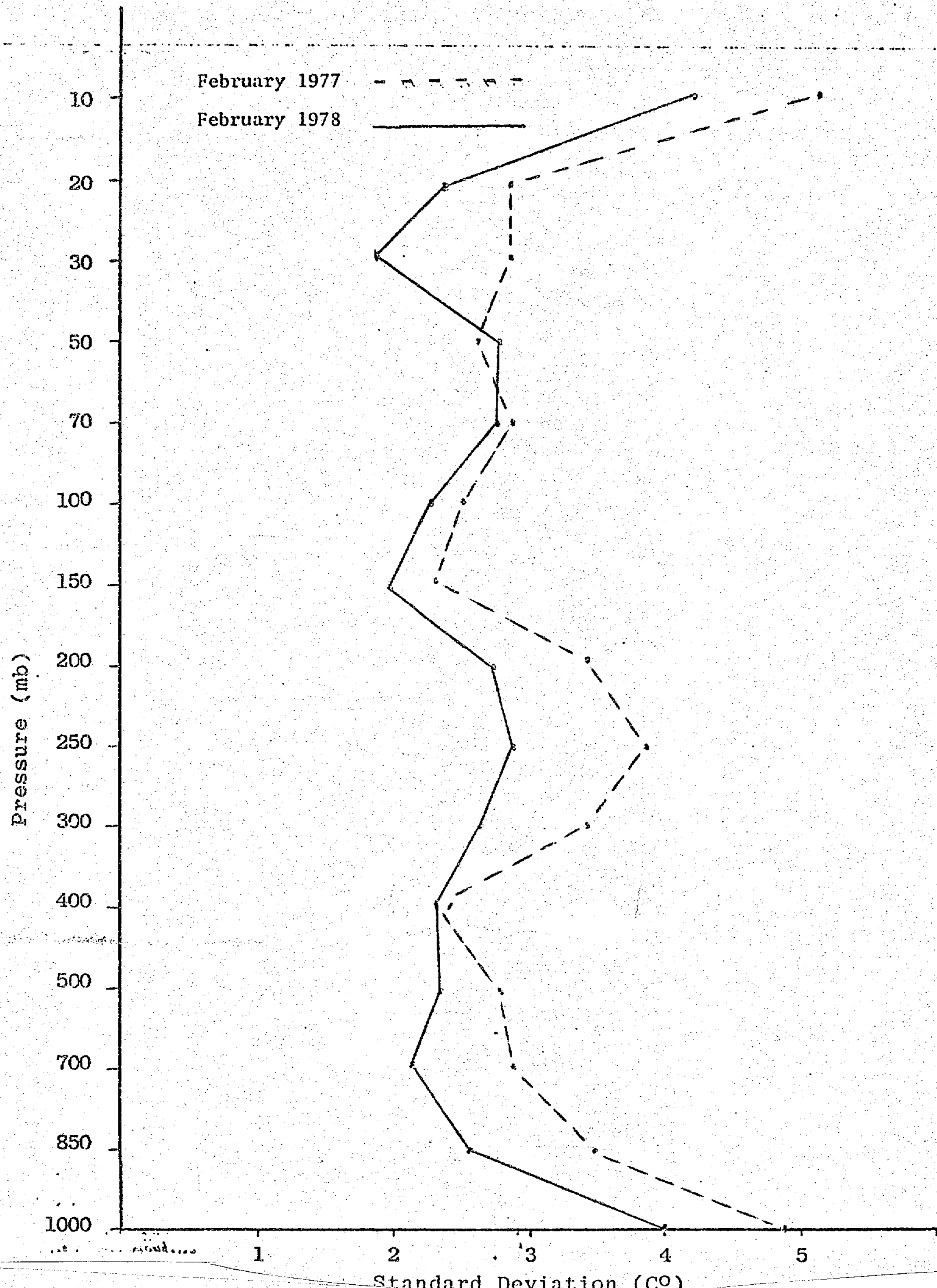

Fig. 1

Standard Deviation of VTPR-Rawinsonde Temperature (C°) Difference for Northern Hemisphere. (Product Systems Branch, National Environmental Satellite Service)

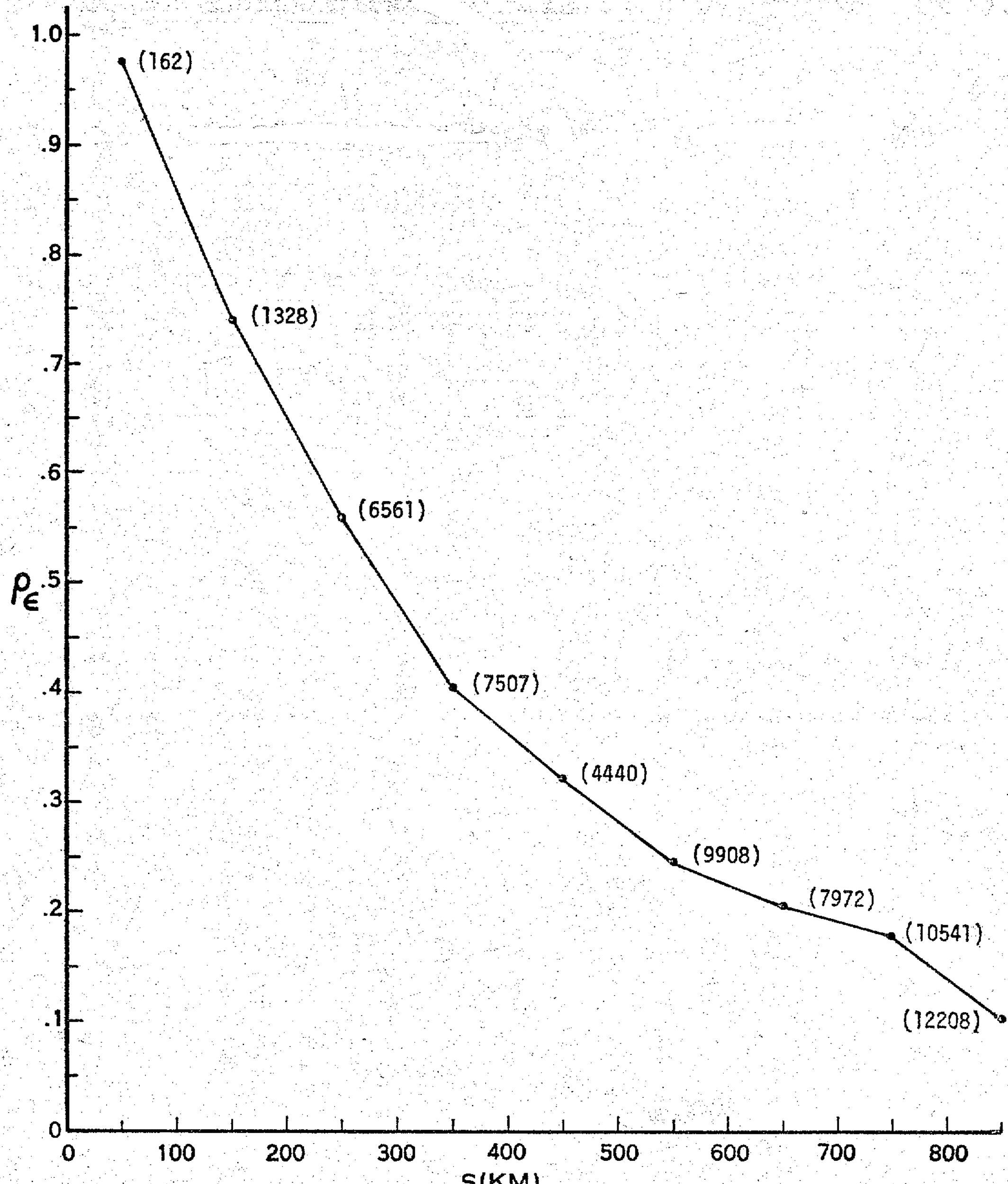

Fig. 2

Isobaric correlation of NIMBUS Satellite temperature errors as a function of observational separation distance s. Correlations for 10 isobaric levels combined, from DST-5 data. (P. Polger and H. Horodeck, Systems Evaluation Branch, NMC.)